Equivalence of the AdS-Metric and the QCD Running Coupling

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Abstract

We use the functional form of the QCD running coupling to modify the conformal metric in AdS/CFT mapping the fifth-dimensional z-coordinate to the energy scale in the four-dimensional QCD. The resulting type-0 string theory in five dimensions is solved with the Nambu-Goto action giving good agreement with the Coulombic and confinement $Q\bar{Q}$ potential.

1 Introduction

The AdS/CFT conjecture relates type IIB superstring theory in the AdS$_5\times$S^5 background with four-dimensional super Yang Mills theory. Supersymmetric QCD is scale invariant with a vanishing β-function. In contrast, QCD has no supersymmetry and a non-vanishing β-function with a well defined running coupling. This defines in our opinion the first task of how to modify the background of supergravity on AdS$_5\times$S^5, in order to obtain a more QCD-like theory: We have to break conformal invariance and disregard supersymmetry. One possible coordinate system of AdS$_5$ is given in the near horizon limit by

$$ds^2_{\text{radial}} = \frac{L^2}{r^2}dr^2 + \frac{r^2}{L^2}(-dt^2 + d\vec{x}^2). \quad (1)$$

In the radial coordinates the boundary of the space is at $r \to \infty$. We are going to use another coordinate system, the Poincaré coordinates, related with the radial coordinates by the transformation $z = L^2/r$. These coordinates are also called conformal coordinates, because one can directly read off the scale invariance of the metric in this coordinate patch

$$ds^2 = G_{\mu\nu}dX^\mu dX^\nu = \frac{L^2}{z^2}(-dt^2 + d\vec{x}^2 + dz^2). \quad (2)$$

L is the radius of AdS$_5$. The boundary of the AdS space is at $z = 0$. A simple ansatz of breaking conformal invariance is to multiply the metric in eq. (2) by the so-called warping function. One can show [1] that global Poincaré invariance demands that the warping function has to depend only on the z-coordinate. Thus the new metric is of the form

$$ds^2_{\text{QCD}} = h(z)\cdot ds^2 = h(z)\frac{L^2}{z^2}(-dt^2 + d\vec{x}^2 + dz^2), \quad (3)$$

where the subscript QCD symbolizes that this is not anymore a metric for the AdS$_5$ space but is a first attempt to obtain a QCD-like theory from this modified AdS$_5$ space.
How should one choose $h(z)$? QCD is a renormalizable quantum field theory. Hence, the UV divergences can be absorbed in a renormalized coupling g. It is common to use the strong coupling constant $\alpha_s = g^2/4\pi$. This coupling is given to the lowest order by the formula

$$\alpha_s(p) = \frac{1}{4\pi\beta_0 \log(p^2/\Lambda^2_{\text{QCD}})} = \frac{4\pi}{(11 - \frac{2}{3}n_f)\log(p^2/\Lambda^2_{\text{QCD}})}. \quad (4)$$

Here p is the scale, which can be chosen to coincide with the transferred momentum [2]. Λ_{QCD} is called the QCD scale parameter, which is to be determined by experiments. Finally $\beta_0 = \frac{1}{(4\pi)^2}(11 - \frac{2}{3}n_f)$ is the absolute value of the first coefficient of the β-function, which is subtraction-scheme-independent. One can see that, for $p \to \infty$, the coupling α_s vanishes, and the theory becomes scale invariant.

It can be shown [3] that the radial coordinate r and thus also the coordinate z corresponds to the energy scale $p \propto 1/z$ of the boundary field theory. Hence, we have for small values of z the UV region of the boundary field theory, and for large values of z we are in the IR region of the boundary field theory. Therefore, the bulk space contains all possible energy scales of the boundary field theory [4]. For QCD correlation functions, it is natural to have a scale-invariant theory in the limit $z \to 0$. In that limit, we should have

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$h(z) \to 1$. Given the fact that the warp factor and the dilaton (which determines the running coupling) are not independent and coupled via the 5D Einstein equations, it seems reasonable to try an ansatz for the warping function that equates it with α_s given in (4). We will see in the rest of the paper that such an ansatz leads to very good agreement with the Cornell potential.

$$h(z) = \frac{c_2}{\log\left[\frac{1}{z^2+l_s^2}\frac{1}{\Lambda^2}\right]}. \tag{5}$$

The introduction of the parameter l_s guarantees the conformal limit $h(z) \to$ finite at $z \to 0$. The requirement $h(z) \to 1$ for $z \to 0$ fixes $c_2 = \log\left(\frac{1}{l_s^2\Lambda^2}\right)$. We assume that Λ is related to the AdS$_5$-radius as $\Lambda = 1/L$ and define the dimensionless parameter ϵ as the ratio

$$\epsilon \equiv \frac{l_s^2}{L^2} = l_s^2\Lambda^2. \tag{6}$$

Then we obtain

$$h(z) = \frac{\log\left(\frac{1}{\epsilon}\right)}{\log\left[\frac{1}{(\Lambda z)^2+\epsilon}\right]}, \tag{7}$$

with the IR singulartiy at

$$z_{\rm IR} = \sqrt{\frac{1-\epsilon}{\Lambda^2}}. \tag{8}$$

According to the equivalence of $1/z$ to energy resolution in four dimensions the scaling factor $h(z)$ indirectly encodes the running behavior of the strong coupling α_s.
In the infrared we have broken the conformal invariance by cutting off the AdS$_5$ space at some finite value of $z = z_{\rm IR}$. In the language of AdS/QCD this set-up is very similar to a hard-wall model [5]. As we will demonstrate the modified coupling naturally incorporates confinement at the infrared energy scale $\Lambda \approx 1/z_{\rm IR}$. In the ultraviolet QCD becomes a scale invariant theory at sufficiently high energies because of asymptotic freedom, which shows up in the metric preserving the conformal form, i.e. $\lim_{z\to 0} h(z) = 1$.

2 Heavy quark potential from AdS/QCD

In QCD one can include quarks as infinitely heavy external probes. To determine the interaction potential $V_{Q\bar{Q}}(R)$ between the quark and the antiquark we use the Wilson loop. The Wilson loop describes the creation of a $Q\bar{Q}$-pair at some time t_1, interaction of the created quark and the antiquark with themselves and the vacuum during a period of time T, and the annihilation of the pair at time t_2. The Wilson loop, c.f. Fig. 1

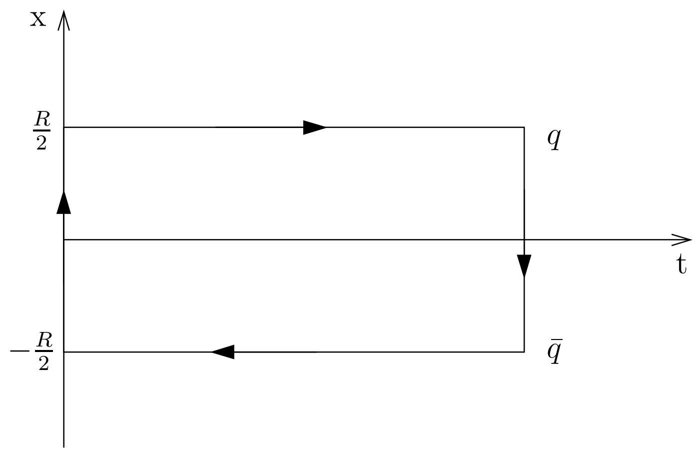

Figure 1: Rectangular Wilson loop contour put on the 4-dimensional boundary of the modified AdS$_5$ space.

is defined as

$$W[C] = \frac{1}{N} Tr P \exp[i \oint_C A_\mu dx^\mu]. \tag{9}$$

The $1/N$-factor is introduced for convenience because there are N terms in the trace over the unit matrix in the fundamental representation of an $SU(N)$ gauge theory, and the P stands for path ordering of the exponential: For $T \to \infty$, the VEV of the Wilson loop behaves as $\langle W(C)\rangle \propto e^{-TV_{Q\bar{Q}}}$.

According to the holographic dictionary [6,7], the expectation value of the Wilson loop in four dimensions should be equal to the string partition function on the modified AdS$_5$ space, with the string worldsheet ending on the contour C at the boundary of AdS$_5$

$$\langle W^{\rm 4d}[C]\rangle = Z^{\rm 5d}_{\rm string}[C] \approx e^{-S_{NG}[C]}. \tag{10}$$

The second relation is obtained by the saddle-point approximation, in which the partition function is just given by the classical action [8]. Hence, we have to consider the classical string worldsheet action S_{NG}. As in the original hadronic string theory the Nambu-Goto action will play a major role to model the gluonic degrees of freedom. However, the string-gauge theory has to be extended to gravity if one looks for a consistent explanation of the metric as a solution of the Einstein equations with a dilaton. This work will be published separately. Note the string worldsheet is embedded into the five-dimensional bulk space. The worldsheet is stretching

from the boundary of AdS_5 at infinity down to a given point resulting in an infinite worldsheet area and thus $\langle W[C]\rangle = 0$. Since our worldsheet is swept out by an infinitely heavy string, the mass of the string times the length of the loop C should be subtracted from S_{NG} [4, 8]. The resulting difference is finite. This is incorporated in the later performed UV renormalization of the Nambu-Goto action.

To calculate the $Q\bar{Q}$ potential we use the AdS/QCD background Euclidean metric

$$ds^2_{Eucl} = G_{\mu\nu} dX^\mu dX^\nu = \frac{h(z)L^2}{z^2}(dt^2 + d\vec{x}^2 + dz^2). \tag{11}$$

The Nambu-Goto action S_{NG} is given by

$$S_{NG} = \frac{1}{2\pi l_s^2} \int d^2\xi \sqrt{\det h_{ab}}, \tag{12}$$

where l_s is the string length and h_{ab} is the induced worldsheet metric: The indices a, b are reserved to the ξ_1, ξ_2-coordinates on the worldsheet, the greek indices μ, ν to the coordinates of the embedding five-dimensional space

$$h_{ab} = G_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\nu}{\partial \xi^b}. \tag{13}$$

In the static gauge, the worldsheet coordinates can be chosen as $\xi^1 = t$ and $\xi^2 = x$. In such a static configuration $z = z(x)$ is the only x-dependent function. The Wilson-loop contour C is located at the boundary of the AdS space, i.e. at $z \to 0$. The set-up is presented in Fig. 1.

The induced worldsheet metric obtained from eq. (11)

$$h_{ab} = \frac{L^2 h(z)}{z^2} \begin{pmatrix} 1 & 0 \\ 0 & 1 + (\frac{\partial z}{\partial x})^2 \end{pmatrix} \tag{14}$$

has to be put into the Nambu-Goto action together with the dimensionless parameter $\epsilon = \frac{l_s^2}{L^2}$ to get

$$S_{NG} = \frac{T}{2\pi\epsilon} \int dx \frac{h(z)}{z^2} \sqrt{1 + (z')^2}, \tag{15}$$

where $z' = \frac{dz}{dx}$ and T comes from the integral over time. Now we can identify

$$\mathscr{L}(z, z') = \frac{h(z)}{z^2} \sqrt{1 + (z')^2} \tag{16}$$

with an effective Lagrangian, and the problem reduces to a simple problem of classical mechanics with the Hamiltonian

$$\mathscr{H} = p(z) \cdot z' - \mathscr{L}, \tag{17}$$

where $p(z) = \frac{\partial \mathscr{L}}{\partial z'}$ is the conjugate momentum. One obtains

$$\mathscr{H} = \frac{-h(z)}{z^2 \sqrt{1 + (z')^2}}. \tag{18}$$

Energy conservation allows one to set $\mathscr{H} = -1/c^2$, where c is a constant

$$\frac{h(z)}{z^2 \sqrt{1 + (z')^2}} = \frac{1}{c^2}. \tag{19}$$

We express this integration constant c via the maximal value of z, which we denote as z_0. Equation (19) yields at $x = 0$:

$$\frac{h(z_0)}{z_0^2} = \frac{1}{c^2}. \tag{20}$$

We can rewrite eq. (19) as:

$$z' = \sqrt{\left(\frac{h(z)c^2}{z^2}\right)^2 - 1}. \tag{21}$$

Using the condition eq. (20) for the maximum and rescaling $z = \nu z_0$, we obtain the inter-quark distance R as a function of z_0:

$$R(z_0) = 2z_0 \int_0^1 d\nu \nu^2 \frac{h(z_0)}{h(\nu z_0)} \frac{1}{\sqrt{1 - \nu^4 \left(\frac{h(z_0)}{h(\nu z_0)}\right)^2}}. \tag{22}$$

By similar transformations we can write the energy, which we get from the Nambu-Goto string action, as a function of z_0

$$V_{Q\bar{Q}}(z_0) = \frac{1}{\pi\epsilon} \frac{1}{z_0} \int_0^1 d\nu \frac{h(\nu z_0)}{\nu^2} \frac{1}{\sqrt{1 - \nu^4 \left(\frac{h(z_0)}{h(\nu z_0)}\right)^2}}. \tag{23}$$

The regularization of the potential is related to the subtraction of the masses of infinitely heavy quarks as discussed before. We subtract the singular part $\propto \frac{1}{\nu^2}$ from the integrand and add its primitive at the upper limit which results in

$$V^{ren.}_{Q\bar{Q}}(z_0) = -\frac{1}{\pi\epsilon} \frac{1}{z_0} + \frac{1}{\pi\epsilon} \frac{1}{z_0} \int_0^1 d\nu \left(\frac{h(\nu z_0)}{\nu^2} \frac{1}{\sqrt{1 - \nu^4 \left(\frac{h(z_0)}{h(\nu z_0)}\right)^2}} - \frac{1}{\nu^2} \right). \tag{24}$$

We continue evaluating eqs. (22) and (23) in terms of the parameter z_0. In order to get a first impression how both integrals depend on z_0, we plot them in Figs. 2 and 3.

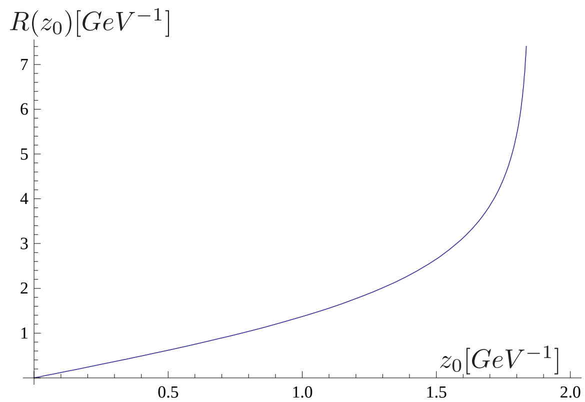

Figure 2: A plot showing $R(z_0)$ for $\epsilon = 0.48$ and $\Lambda = 0.264\,\mathrm{GeV}$.

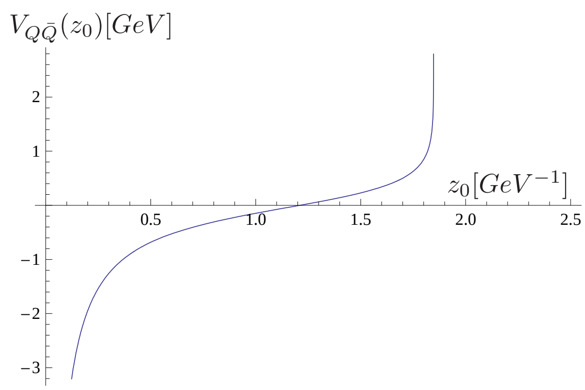

Figure 3: A plot showing $V_{Q\bar{Q}}(z_0)$ for $\epsilon = 0.48$ and $\Lambda = 0.264\,\mathrm{GeV}$.

The phenomenological Cornell potential of the form $V_{Q\bar{Q}} = -\frac{a}{R} + \sigma R$ determines the parameters in the underlying metric. We fix the dimensionless parameter ϵ to the parameter a in the Coulombic part of the potential and the parameter Λ to the string tension σ in the long distance $Q\bar{Q}$-interaction. It is quite natural to have two parameters in the metric to determine two parameters in the potential. The agreement with the phenomenological potential can be improved [9]. Indeed the found Λ will be similar to $\Lambda_{\overline{\mathrm{MS}}}$ in QCD. The QCD string has been the origin of hadronic string theory which has been supported by lattice simulations of QCD where one identifies the stretched tube of the color electric flux with a string. We can see from Fig. 2 that the $Q\bar{Q}$ distance R depends linearly on z_0 for small z_0. Looking at the definition of $h(z)$ in eq. (7) we realize that, for $z_0 \approx 0$, the ν-dependence of $h(z)$ is suppressed. Hence, we only make a negligible error when performing Taylor expansion of the integrand of $R(z_0)$ in eqs. (22) and (23) at $z_0 = 0$ up to the first order and then integrating over ν in order to obtain the behavior of the potential $V_{Q\bar{Q}}$ at small $Q\bar{Q}$-separations R. For eq. (22) this yields

$$R(z_0) = 2\sqrt{\pi}\frac{\Gamma(3/4)}{\Gamma(1/4)} z_0 + \mathcal{O}(z_0^2), \qquad (25)$$

which is exactly the result we would have obtained in the conformal case with $h(z) = 1$.
We expand the integrand of eq. (23) at $z_0 = 0$ to the order $\mathcal{O}(z_0^2)$, integrate over ν and then insert $z_0(R)$ from eq. (25), and finally obtain

$$V_{Q\bar{Q}}(R) = -2\left(\frac{\Gamma(3/4)}{\Gamma(1/4)}\right)^2 \frac{1}{\epsilon R} + \sigma R, \qquad (26)$$

with

$$\begin{aligned}\sigma &= \left(-\frac{1}{4\pi} + \frac{27}{256\pi}\frac{\Gamma(1/4)^2}{\Gamma(7/4)^2}\right)\frac{\Lambda^2}{\epsilon^2\log(1/\epsilon)} \\ &\approx 0.443\frac{\Lambda^2}{\epsilon^2\log(1/\epsilon)}.\end{aligned} \qquad (27)$$

We can see from the $1/R$-term in eq. (26) that it is exactly the same as in the case of the conformal metric [8]. In order to adjust the above potential to the value of the Coulombic part of the $Q\bar{Q}$ interaction given in [10], namely $V_{Q\bar{Q}} = -\frac{a}{R} + \sigma R$ with $a = 0.48$ and $+\sigma = 0.183\,\mathrm{GeV}^2$ we have to choose

$$\epsilon = 0.48 \qquad (28)$$
$$\Lambda = 264\,\mathrm{MeV}. \qquad (29)$$

This result looks rather reasonable. For example, the value of the scale parameter in four-flavor QCD[1] is $\Lambda_{\mathrm{QCD}}^{n_f=4} = 274 \pm 30\,\mathrm{MeV}$ [2]. Having fixed the two parameters we can now numerically evaluate the heavy quark potential to test the form on all length scales. Fitting the numerical potential plotted in the interval $R \in [0.1\,\mathrm{GeV}^{-1}, 9.6\,\mathrm{GeV}^{-1}]$ in Fig. 4 to a Cornell-like potential $V_{Cornell}(R) = -\frac{a}{R} + \sigma R$ yields $a = 0.47$, $\sigma = 0.181\,\mathrm{GeV}^2$. If one takes the dependence of these parameters on the fit intervall into account the numerically determined values coincide with the analytical ones.

For further applications it is important to note that the validity of a gravity dual to the string description is $\frac{L^4}{l_s^4} \gg 1$. We obtain $\frac{L^4}{l_s^4} \approx 4.3$. This choice is

[1] We do not want to compare exactly to four-flavor QCD, but want to show that Λ has the correct magnitude.

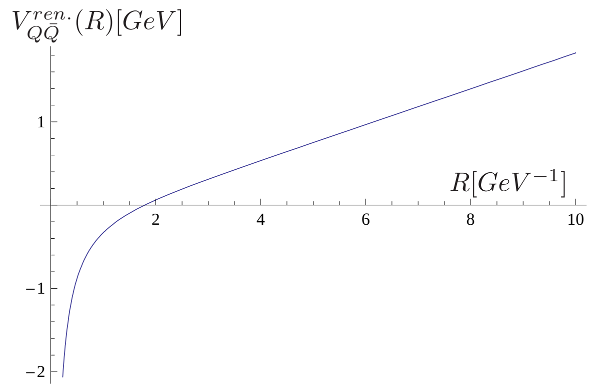

Figure 4: Numerically calculated heavy-quark potential for the modified metric, eq. (11), using $h(z)$ from eq. (7).

imposed by the $Q\bar{Q}$ potential. So it may be necessary to include higher correction to the simple form of gravity with an AdS-negative cosmological constant. One can try to include these corrections via a modified dilaton potential in the corresponding five-dimensional gravity theory. Due to the form of the warp factor, the resulting dilaton dynamics may only reproduce the β-function of QCD approximately. [11] In perturbative QCD one knows that for $R \to 0$ the potential $V_{Q\bar{Q}} = -\frac{N_c^2-1}{2N_c}\alpha_s \frac{1}{R}$ and hence scales with $g_{\rm YM}^2 N_c$. It is possible to show that $\frac{L^4}{l_s^4} = g_{\rm YM}^2 N_c$ [4]. The potential obtained, eq. (26), is only proportional to $\frac{L^2}{l_s^2}$ and hence scales with $g_{\rm YM}\sqrt{N_c}$. An explanation of this discrepancy may be that the limit $l_s \to 0$ corresponds to a Yang-Mills theory with a strong coupling $g_{\rm YM}^2 N_c$. For realistic AdS/QCD this argument has to be studied in more detail. An interesting result of our calculation is the proportionality between σ and Λ given by eq. (27), once ϵ is fixed.

From the taylor expansion in eq. (30) one can see that there exists a complex singularity at $z_0 = z_\gamma$. Let us determine z_γ. From eq. (22), one can see that the dominant contribution to the integral arises at $\nu = 1$. A Taylor expansion of the integrand at $\nu = 1$ followed by the integration yields up to $\mathcal{O}(1-\nu)$ the following expression

$$R(z_0) = \frac{2iz_0}{\sqrt{-1 + \frac{z_0^2\Lambda^2}{(\epsilon + z_0^2\Lambda^2)\,\mathrm{Log}\left[\frac{1}{\epsilon + z_0^2\Lambda^2}\right]}}}, \tag{30}$$

which is only real for

$$\frac{z_0^2\Lambda^2}{(\epsilon + z_0^2\Lambda^2)\,\mathrm{Log}\left[\frac{1}{\epsilon + z_0^2\Lambda^2}\right]} < 1. \tag{31}$$

This inequality can be solved in terms of the ProductLog function[2], and one obtains for the above determined parameters

$$z_0 < z_\gamma \tag{32}$$

$$z_\gamma = \sqrt{\frac{\epsilon}{\Lambda^2}\left(\frac{1}{\mathrm{ProductLog}(\epsilon e)} - 1\right)} \approx 1.85\,\mathrm{GeV}^{-1}, \tag{33}$$

where the $e \approx 2.71828$ in the denominator is the base of the natural logarithm. A similar analysis of the integral in eq. (23) yields the same complex singularity at $z_0 = z_\gamma$.
This singularity defines a horizon[3] in contrast to the purely conformal AdS background, where there is no upper bound on the parameter z_0. For larger R-values, one obtains a larger z_0. In case of the modified metric, the z_0 of the worldsheet is limited by the horizon. For other confining backgrounds based on the running coupling we refer to the studies of Kiritsis et al. in [12, 13]. In these papers, the authors analyze various confining backgrounds by studying the long-range part of the $Q\bar{Q}$-potential, given in the form derived in [14]. It should be mentioned that our background given by $h(z)$ of eq. (7) satisfies the criterium for a confining background by eq. (3.12) of Ref. [13]. We also refer to [15], where the Cornell potential is derived in various backgrounds.

Finally we show the minimal worldsheets for the modified metric given by eq. (11) in Fig. 5.

One can see that the worldsheets are flattening with the increase of the $Q\bar{Q}$-separation R. There exists one crucial difference compared to the conformal case, namely that we have an upper bound $z_\gamma \approx 1.85\,\mathrm{GeV}^{-1}$, dictated by the complex singularity discussed above. An upper bound on z_0 implies an upper bound for R. For the parameter values $\epsilon = 0.48$ and $\Lambda = 0.264\,\mathrm{GeV}$, we obtain for the upper bound $R \approx 2.8\,\mathrm{fm}$. We can see from Fig. 5 that the worldsheet already reaches its maximal z-value for x away from zero, and the surface becomes completely flat, which makes a confining potential. One can interpret this effect as touching the IR horizon. Dubovsky and Rubakov [16] obtain a quite similar behavior, while studying two charges in front of a brane. There, the electric flux of the charges drops down on the brane, while having still confinement.

[2] ProductLog(f) gives the principal solution for w in $f = we^w$.

[3] This horizon should not be confused with the IR singularity of the modified metric at $z_{\rm IR} \approx 2.73\,\mathrm{GeV}^{-1}$.

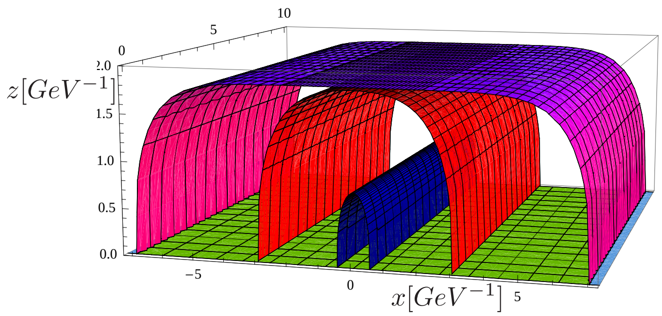

Figure 5: Nambu-Goto worldsheet for the non-conformal metric of eq. (11) at the values of the $Q\bar{Q}$-separation $R = 1\,\mathrm{GeV}^{-1}$ (blue), $R = 6\,\mathrm{GeV}^{-1}$ (red), and $R = 14\,\mathrm{GeV}^{-1}$ (magenta) plotted over the Wilson loop area (green) in the (x, t) plane.

We do not have string-breaking effects, despite having a maximal $Q\bar{Q}$-separation $R \approx 14.1\,\mathrm{GeV}^{-1}$. This is due to the fact that we have no dynamical quarks.

3 Conclusion

We have chosen the functional form of the warping factor of the AdS-metric eq. (7) such that it coincides with the functional form of the QCD running coupling eq. (4). Using the holographic dictionary [6–8] we then extract the $Q\bar{Q}$-potential. Only one new parameter $\epsilon = 0.48$ has to be fitted to reproduce the short and long range Cornell potential [10]. The other parameter $\Lambda = 264\,\mathrm{MeV}$ in the metric is close to $\Lambda_{\mathrm{QCD}}^{n_f=4}$. The phenomenology of equivalence proves to be successful and simple. Comparing with one of the previous calculations [9] of the $Q\bar{Q}$-potential one sees in Tab. 1 that our work can reproduce both the strength of the Coulomb interaction a and the string tension σ at the same time. The parameter $L^4/l_s^4 \gg 1$ indicates that the string theory has a meaningful gravity approximation which we will present in a separate paper [11].

Acknowledgement

We thank D. Antonov for useful discussions.

	[9]	our work	Cornell [10]
L^4/l_s^4	0.89	4.34	-
a	0.22	0.47	0.48
σ	$0.18\,\mathrm{GeV}^2$	$0.18\,\mathrm{GeV}^2$	$0.18\,\mathrm{GeV}^2$

Table 1: Comparison with [9].

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